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Optimization and Analysis of Multi-beam System in Marine Mapping

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Abstract

The title of this paper is based on question B of Chinese College Students in 2023. It aims to derive the calculation formula of seawater depth, coverage width, and overlap rate between adjacent strips. The study explores the influence of the β direction angle of different measuring lines on coverage width and designs a group of measuring lines that can measure the shortest length and fully cover the sea area to be tested. With the advancement of China's marine development strategy, the acquisition of submarine topographic data, and the design and optimization of multi-beam sounding systems are crucial. Through mathematical derivation and geometric analysis, this paper successfully establishes a computational model of seawater depth, coverage width, and overlap rate, providing a practical theoretical basis and technical support. These research results are significant for improving the efficiency and quality of marine resources development and environmental protection work. They also offer a useful reference for the progress and application of marine surveying and mapping technology.

Keywords

Mathematical modeling, multi-beam sounding system, system optimization, measuring line design

1. Introduction

The topic of this paper is based on question B of Chinese College Students in 2023. It aims to derive and establish the calculation formula of seawater depth, coverage width, and overlap rate between adjacent strips. At the same time, we explore the influence of β on coverage width and design a set of measurement lines with the shortest measurement length that can fully cover the sea area to be tested. In the field of Marine mapping, the acquisition of seabed topography data and the design optimization of multi-beam sounding systems are crucial. With the promotion of China's Marine development strategy, island development has become a basic project, and the seabed topography and landform data are the key support. Although the development of multi-beam systems improves measurement precision and efficiency, their complex structure and data processing challenges remain to be addressed. Therefore, this paper aims to optimize the line design in the multi-beam system to improve the efficiency and quality of data acquisition, provide reliable support for Marine resource development and environmental protection, and promote the progress of Marine mapping technology.

2. Model assumptions and symbolic instructions

(1) If the slope is smooth, that is, there is no significant tilt and no significant increase or decrease.
(2) Assuming that the sea surface in the tested area is calm, the sway reading of the measuring ship is about 5 ° to ensure the accuracy of multi-beam measurement

And coverage width data are accurate.

(3) It is assumed that water temperature factors do not affect the multibeam measurement coverage width.

Table 1. Shows the symbol description

symbol	explain	unit
h_0	Sea water depth at the central point of the sea area	m
w_0	The coverage width at the center point of the sea area	m
α	angle of slope	linear measure
d	Measure the line spacing	m
d'	The distance between the survey line and the slope surface and the sea surface intersection lines	m
w'	Part of the coverage width of the projection	
w''	Part of the coverage width of the projection	
w_0'	A partial projection of the cover width at the central point of the sea area	
w_0''	A partial projection of the cover width at the central point of the sea area	
w	Cover width	
w_0	Cover the width at the center point of the sea area	
γ	The angle between the cover width and the horizontal plane	linear measure
β	path angle	linear measure
θ	Multi-beam transducer opening angle	linear measure
η	Overlap rate of adjacent test lines	

3. The calculation formula of seawater depth and the formula of overlap rate between adjacent bands are derived

Establish a two-dimensional coordinate system (see Figure 1), the x-axis represents the distance of each measurement line from the central point, a total of nine measurement lines, so the x value is {-800, -600, -400, -200,0,0,200,400,400,600,800}, the origin of the x-axis is set as the sea center point, and the y-axis indicates the seawater depth.

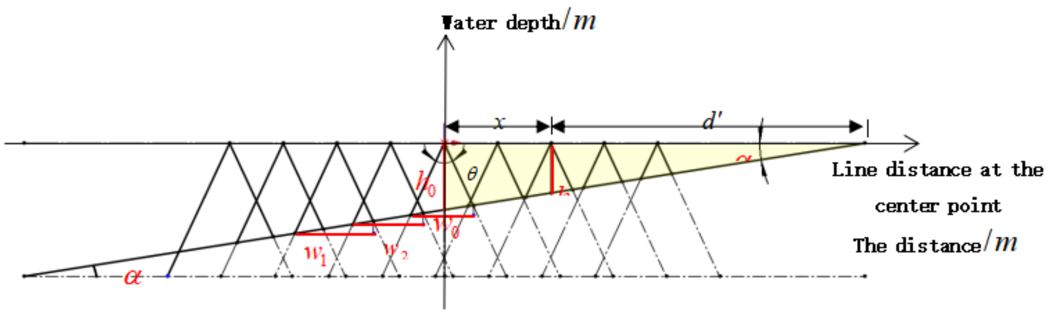

Figure 1. Layout of multiple sets of test lines.

3.1 Formulation of two-dimensional plane seawater depth

Since the slope extension line and the axis will be crossed at one point, let the intersection and the interval distance be a series of similar triangles with the corresponding composition [1], we will discuss this series of similar triangles. Since x is positive and negative, in the formula deduction of seawater depth, x> 0 and x <0 will be discussed, and the following formula is derived:

(1) When x> 0:

$$\begin{cases} \dfrac{h}{d'} = \tan\alpha \\ \dfrac{h_0}{x+d'} = \tan\alpha \end{cases} \tag{3-1}$$

Conjunctive solution:

$$h = -x\tan\alpha + h_0 \tag{3-2}$$

(2) When the x is <0:

$$\begin{cases} \dfrac{h}{-x+d'} = \tan\alpha \\ \dfrac{h_0}{d'} = \tan\alpha \end{cases} \tag{3-3}$$

Conjunctive solution:

$$h = -x\tan\alpha + h_0 \tag{3-4}$$

The classification discussion results show that the seawater depth formula of x> 0 and x <0 is the same, so the seawater depth formula when the seabed terrain is not flat is [2]:

$$h = -x\tan\alpha + h_0 \tag{3-5}$$

3.2 The derivation of the formula for the 2 D plane coverage width

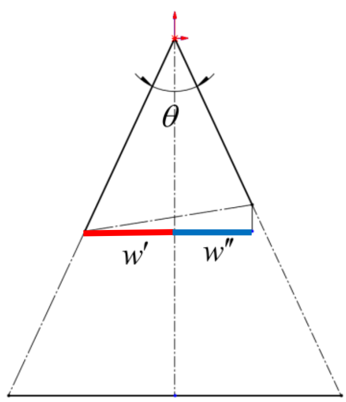

Figure 2. Schematic diagram of the two-dimensional plane of the ocean sounding.

A similar triangle corresponds to the ratio of the side length equal to the ratio of the high line, so:

$$\begin{cases} \dfrac{h_0}{h} = \dfrac{w_0{}'}{w'} \\ \dfrac{h_0}{h} = \dfrac{w_0{}''}{w''} \end{cases} \tag{3-6}$$

$$\begin{cases} w = w' + w'' \\ w_{0=} w_0' + w_0'' \end{cases} \tag{3-7}$$

United stand and get: (1-6) (1-7)

$$w = \frac{h}{h_0} w_0 \tag{3-8}$$

3.3 Formula derived for the overlap rate between adjacent bands in 2 D planes

$$w_{n-1}'' = \frac{w_0'' \cdot h_{n-1}}{h_0} \tag{3-9}$$

$$w_n'' = \frac{w_0' \cdot h_n}{h_0} \tag{3-10}$$

$$\eta = \frac{w_{n-1}'' + w_n' - d}{w_{n-1}' + w_{n-1}''} \tag{3-11}$$

4. Effect of different line angles on the coverage width of the sounding system

In this part, the influence of heading Angle and coverage width should be established. The research method is the control variable method, that is, the same measurement line in a certain direction (with a certain control), the independent variable is the distance between the ship and the central point of the sea area, and the dependent variable is the coverage width (as shown in the figure). Question 1 is fixed (90° or 180°), while in this part, with different values, that is, {0,45,90,135,180,225,270,315}, it will rise from the 2 D plane to the 3 D plane. Here is a detailed demonstration of the coverage width formula [3]:

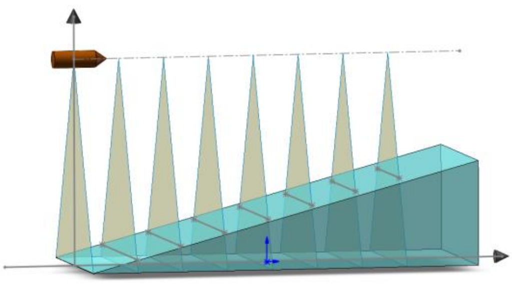

Figure 3. The 3 D schematic diagram of the sounding system.

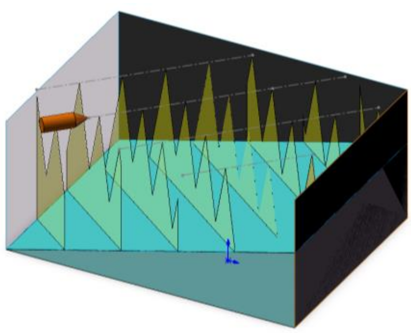

Figure 4. 3 D schematic of rotation state.

Because the slope Angle is a fixed value, the seawater depth at the fixed position is also certain. So, for different heading Angle can be imagined by the isosceles triangle rotation of a cone (Figure 5), is any section of the transducer emission multiple beam (section for isosceles triangle, top Angle for the transducer open Angle), slope and beam surface intersection can be understood with different angles of the cone cross line projection on the horizontal plane [4]. According to the solution in the third chapter, the formula of seawater depth is that the depth of the sea from the ship to the central point of the sea area. θ $h = -x \tan\alpha + h_0$

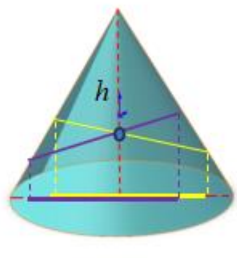

Figure 5. Schematic representation of the beam coverage at different heading angles.

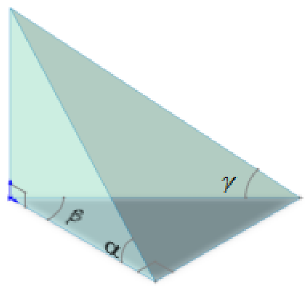

Figure 6. Schematic diagram of the multibeam emission of the sounding system transducer.

Based on the knowledge of solid geometry, it can be deduced that:

$$x = -d' \cos\beta \tag{4-1}$$

According to the triangle angles (3-7):

$$\begin{cases} w_0' = \dfrac{h_0}{\sin\left(\dfrac{\pi}{2} - \alpha - \dfrac{\theta}{2}\right)} \bullet \sin\dfrac{\pi}{2} \bullet \cos\alpha \\ w_0'' = \dfrac{h_0}{\sin\left(\dfrac{\pi}{2} + \alpha - \dfrac{\theta}{2}\right)} \bullet \sin\dfrac{\pi}{2} \bullet \cos\alpha \end{cases} \tag{4-2}$$

Joint (4-2) and (3-6):

$$w = \sin\frac{\theta}{2}\cos\gamma\left(\frac{h}{\sin\left(\dfrac{\pi}{2} - \gamma - \dfrac{\theta}{2}\right)} + \frac{h}{\sin\left(\dfrac{\pi}{2} + \gamma - \dfrac{\theta}{2}\right)}\right) \tag{4-3}$$

β $\beta = 0°$ $\beta = 180°$ β 2π

We can classify and integrate different ones according to the law. We found that when or when the line moves along the depth line, when the value is equal, just the same line, but in the opposite direction, respectively from deep water to shallow water [5], and from shallow water to deep water. The following will establish, and discuss the relationships in the solid geometry, and get the expressions: γ α β γ

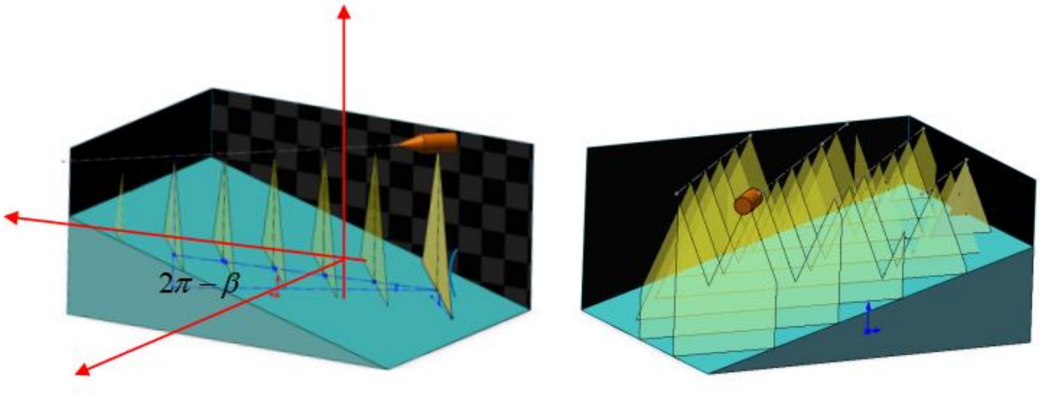

Figure 7. Multibeam-sounding simulation diagram.

According to the geometric relations, the available

$$\tan\gamma = \left|\tan\alpha\sin\beta\right| \tag{4-4}$$

With (3-7) formula, together, can obtain:

$$w = \frac{2\left(d'\cos\beta\tan\alpha + h_0\right)\sin\theta}{\cos\theta + \left(\cos\theta - 1\right)\left(\tan^2\alpha\sin^2\beta\right) + 1} \tag{4-5}$$

5. Effect of different line angles on the coverage width of the sounding system

This part considers a rectangular sea area, 2 nautical miles long from north to south and 4 nautical miles wide from east to west. The seawater depth at the center point of the sea area is 110 meters, showing the characteristics of deep west and shallow east, with a slope of 1.5°. The multi-beam transducer was opened at 120°. A set of lines is designed to achieve the shortest measurement length while completely covering the entire area to be tested and ensure that the overlap between adjacent bands is between 10% and 20%.

5.1 The sounding situation is optimal when the parallel isometric line is measured

When the measuring line is parallel to the Shen depth line (*i.e.*), the measurement line is the longest, the total length is the shortest, and the optimal depth is reached when the measuring line is parallel to each other. The following proof is given from a mathematical perspective:

First, the top view of the design scenario is shown in the figure:

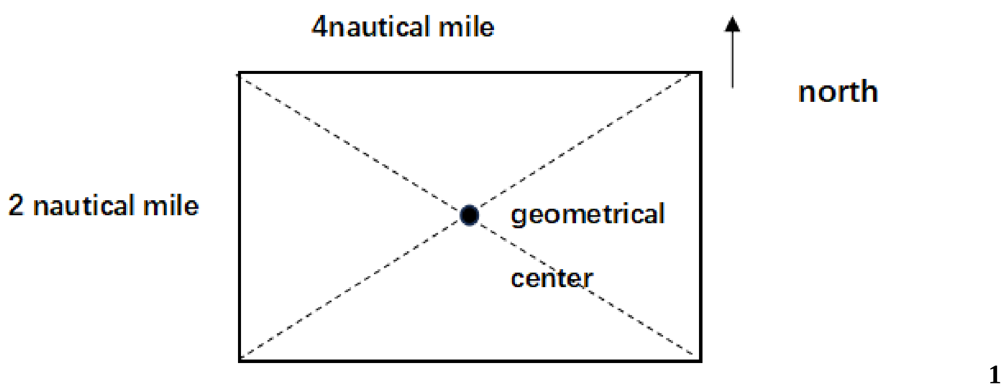

Figure 8. Top view of the applied sea area.

A straight line is perpendicular to the normal direction, and a route is determined through the geometric center. Because the geometric center point has been given in the scene, the seawater depth is 110m, that is, the problem can be simple through this point. Rotate β angle and extend the side of the rectangular sea, the coverage width of the route is a trapezoid. Since this design should make the whole sea area completely covered, so a survey line is determined, and the route distance between the survey line and the route passing through the geometric center point should not exceed the coverage width at the highest point.

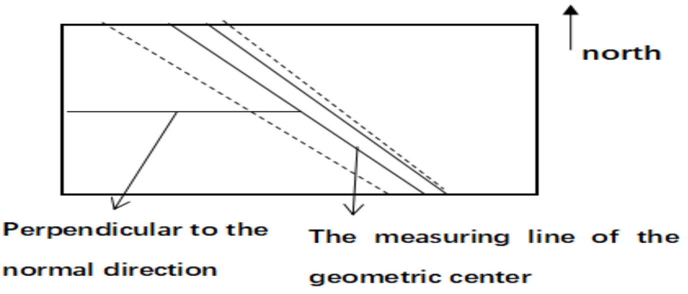

Figure 9. Schematic diagram of the line measurement situation.

The section at the highest point is shown:

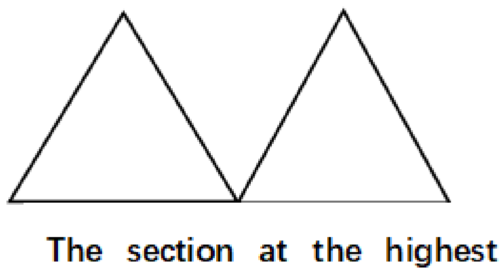

Figure 10. The highest section of the sea area.

So as to ensure that the whole sea area can be covered, if leaving the highest point, there will be leakage, does not meet the design requirements. The larger the spacing between the measurement lines, the measurement line is sparse, that is, the total length of the measurement line is the smallest. At β> 90°, the smaller the β Angle, the larger the spacing between the measurement lines, the larger the adjustable distance of the measurement line, and the smaller the total length of the measurement line. At β <90°, the greater the β Angle, the spacing between the two lines. In summary, at β =90°, the total length of the line is smaller, so that the optimal line can be obtained.

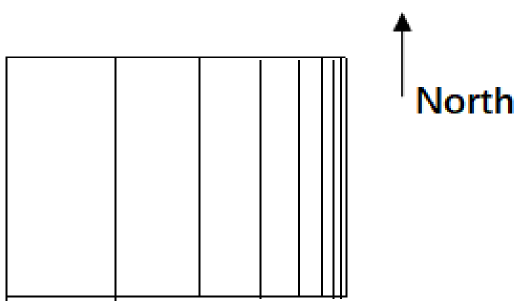

Figure 11. Schematic diagram of measuring line density at different heading angles.

When the measurement line is parallel, the coverage width will be maximized. The larger the spacing d between the two parallel measurement lines is, that is, the larger the coverage width is. However, according to the design requirements, the overlap rate should be 10%, that is, the overlap rate is 10% optimal. The elevation view of the seabed topography is shown in Fig. 12.

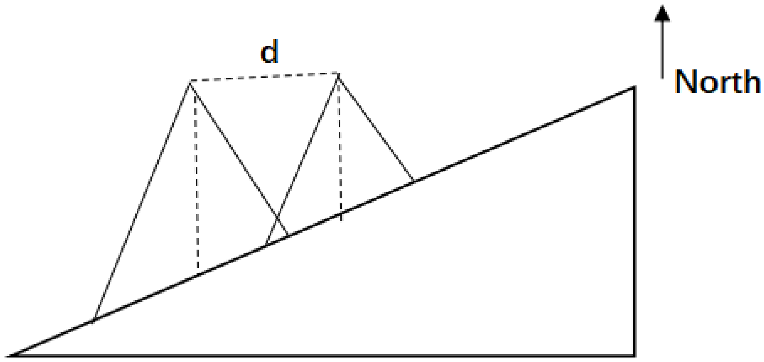

Figure 12. The front view of the seabed topography.

5.2 Constraints

According to the above proof, in order to reasonably design the measuring line, the design requirements for the measuring line design are as follows: (1) the measurement line is as large as possible and the measurement length is as short as possible; (2) completely covers the whole sea area to be tested; (3) the overlap rate between adjacent strips meets 10%-20%.

For (1), in order to make the measurement line data as large as possible and the measurement length as short as possible, it has been proved in detail in 5. $\beta = 90°$ .1

For (2), set the number of measurement lines as, then make the measurement length as short as possible, that is, to meet as small as possible. The line will be parallel to the shorter side of the seaside, that is, the line is a group of parallel lines parallel to the length of 2 nautical miles. The schematic diagram is as follows:

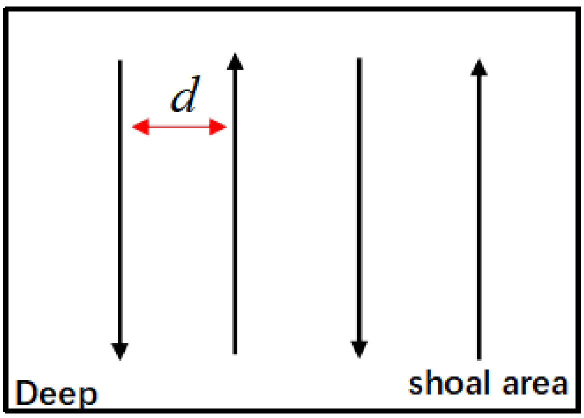

Figure 13. Schematic diagram of the measuring line layout.

According to the calculation formula of the rectangular sea area:

$$N = \frac{4}{d} \tag{5-1}$$

It refers to the evidence between the test lines. Then, if the strip wants to completely cover the whole area to be tested, that is, d is integrated as large as possible to ensure the full coverage of the area.

For (3), the overlap rate formula (3-10) is derived in Chapter 3, and the overlap rate is satisfied between 10% and 20%. In order to make the calculation simple and meet the requirements (1), the measurement line is arranged parallel from the west to the east. In this case, the space between the measuring line is as large as possible and the measurement length is as short as possible. Therefore, for this design, the fixed value is 10%.

5.3 Formula derivation

Establish coordinate system:

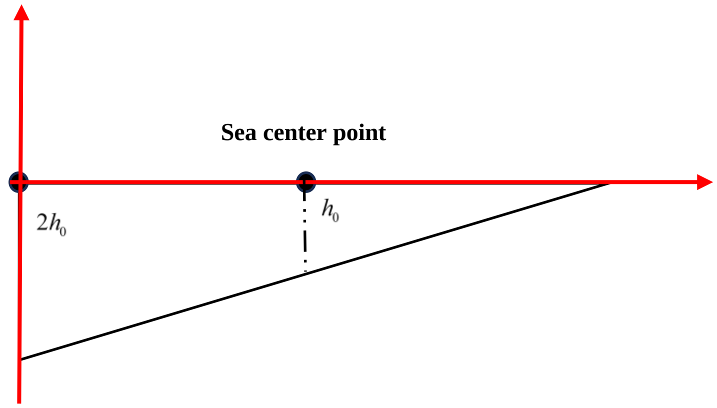

Figure 14. Schematic diagram of the coordinate system

There is a measuring line between the measuring line and the measuring line in the central sea area. The distance between the two measuring lines is, so the coordinates of the measuring line are obtained: nd n

$$x_n = x_0 + nd \tag{5-2}$$

$$x_0 = \frac{2\cos\gamma \sin\frac{\theta}{2} h_0}{2\cos\left(\gamma + \frac{\theta}{2}\right) + \cos\gamma \sin\frac{\theta}{2} h_0} \tag{5-3}$$

$$h = \frac{4 - x_0 - nd}{2} h_0 \tag{5-4}$$

From the similar of similar and combination (3-7):

$$\begin{cases} w'_{n+1} = \dfrac{h_{n+1}}{h_0} w_0{}' \\ w''_n = \dfrac{h_n}{h_0} w_0{}'' \end{cases} \tag{5-5}$$

Settings, bring in (3-10) available at:

$$\eta = \frac{w''_{n-1} + w'_n - d}{w'_{n-1} + w''_{n-1}} = 0.01 \tag{5-6}$$

Joint vertical (3-2), (4-1), (5-5) can obtain the recursive function relation:

$$(-(x_n + d)\tan\alpha + h_0)w'_0 + (-x_n \tan\alpha + h_0)w''_0 - h_0 \bullet d = 0.1 \bullet (-x_n \tan\alpha + h_0) \bullet w_0 \tag{5-7}$$

According to (5-27), recursively calculate each d value, through the functional relation between the x coordinate value and d:

$$x_n = x_{n-1} + d \tag{5-8}$$

Further calculate each d value recursively to complete the design of multiple beam measurement lines. x x_0 x_n d From west to east, it is the distance between a certain test line and the previous test line, for example, between and corresponding. The following is a schematic diagram of the test line layout: x_0 x_1 d_1

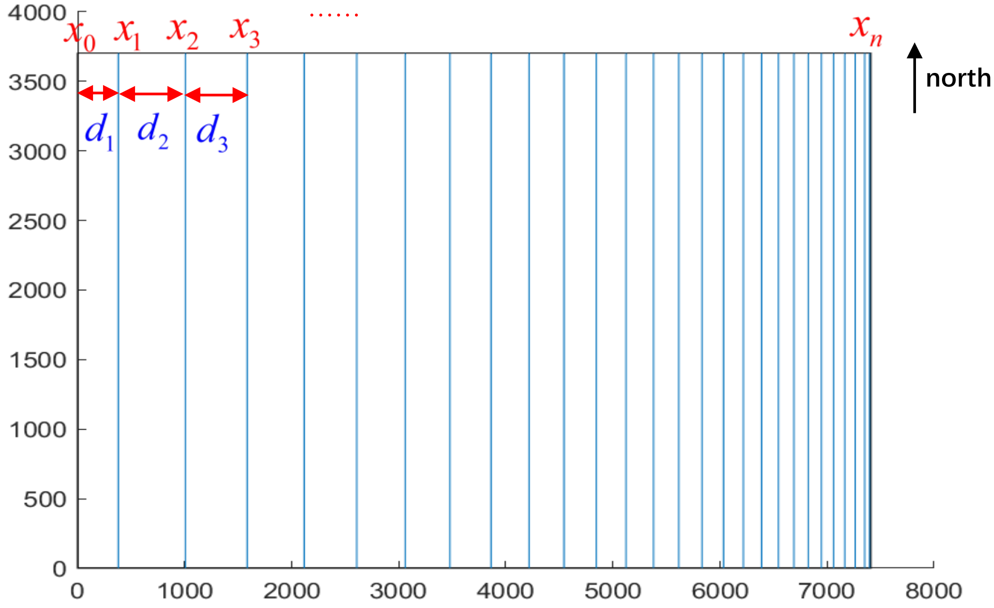

Figure 15. Schematic diagram of the wiring design in the situational sea area.

6. Conclusion

In conclusion, this study successfully derived the calculation formula of seawater depth, coverage width and overlap rate between adjacent bands, revealed the influence of different line direction angles β on the coverage width, and designed a group of measurement line layout schemes that can measure the shortest length and fully cover the sea area to be tested. These achievements provide practical theoretical support and technical guidance for the field of Marine surveying and mapping, help to improve the efficiency and quality of Marine resources development and environmental protection work, and promote the progress and application of Marine surveying and mapping technology.

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