



A Study on the Influence of Wood's Roughness on the Stability of Wooden Arch Bridges

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Abstract

Without any iron pieces in the structure, there are fabulous principles behind the ancient arch bridges in China, giving people a chance to comprehend the wisdom of ancient architecture and ancient people. This study mainly focuses on the influence of wood's roughness on the stability of wooden arch bridges. The paper explains structural engineering theories in physical and mathematical ways, exploring the stability of the bridge by simplifying it to a physics model and through physical analysis. Additionally, two computer programs were used to visualize the relationship between different variables. The paper deduces how to make the bridges more stable step by step. The calculations and programs in this paper will be applicable under various circumstances. The aim of this paper is to raise awareness worldwide about protecting this amazing treasure and provide some practical theoretical advice for architecture to revive wooden arch bridges in the future.

Keywords

Structural Engineering; Roughness; Stability; Wooden Arch Bridges; Intangible Cultural Heritage

1. Introduction

The ancient wooden arch bridge is a legacy of the historical and cultural heritage of China. However, most of them were destroyed for unknown reasons. The theories behind the wooden arch bridges are so awesome and will be illustrated in this paper.

2. Main Text

2.1 The history of the wooden arch bridges

The painting "Ascending the River at Qingming Festival", shown in Figure 1, by Zeduan Zhang of the Northern Song Dynasty, which is still preserved in the Palace Museum, depicts the clever construction of the wooden structure of the Hongqiao Bridge on the Bianshui River during the Song Dynasty and the bustling scene of pedestrians. According to Yuanlao Meng of the Song Dynasty in his "A Dream of Splendor", "Seven miles from the east water gate is called the Hongqiao Bridge. The bridge has no pillars and is made of huge wooden frames, decorated with alchemy, like a flying rainbow.". This "pillarless Hongqiao", built with a huge wooden frame, is academically known as a wooden arch bridge. According to "The Compilation of Song's Regulations" and "Song History", it solves the problem of frequent collisions between boats and piers on the Bian River and has become a model of famous bridges in China for nearly a thousand years [1-3].

Wooden arch bridges not only provide transportation functions but are also commonly landmark buildings in rural areas. The corridors and houses of bridges in various regions are generally equipped with stages, benches, stalls, etc., which are important places for villagers to drink tea, cool off, watch plays, gather, rest, watch, trade, and other activities. Every year during the Dragon Boat Festival, Chinese Valentine's Day, and other festivals, people hold folk activities such

as “walking on the bridge” on the wooden arch bridge. This enables the wooden arch bridge to function as a community public cultural space, becoming an important cultural symbol and link that consolidates the emotions of community residents and maintains local cultural identity.

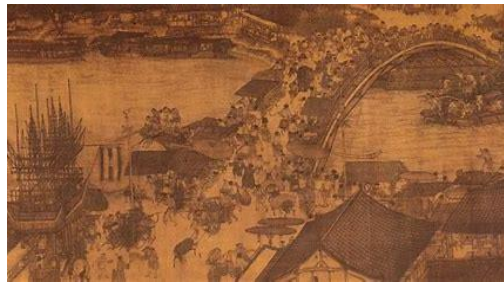


Figure 1. The painting "Ascending the River at Qingming Festival".

Wooden arch bridge is a traditional wooden structure bridge widely used in mountainous areas of Fujian and Zhejiang, China. This type of Hongqiao technology, which was popular in the Northern Song Dynasty, is the tail end of China's ancient bridge-building techniques and the last rare living fossil in the history of bridge construction in China. In the context of modern civilization, the identity of wooden arched bridges as a means of transportation has declined. However, this beautiful building, like a rainbow drinking stream, has long corridors on the bridge that can shelter from wind and rain, market trade, and worship gods. People spend every grand festival on the funded construction of the corridor bridge, praying for the peace of the world and remembering their own joyful moments [4, 5].

In 2009, the construction techniques of Chinese wooden arch bridges were included in the UNESCO List of Intangible Cultural Heritage that urgently needed protection. The government and local people took a series of protection measures, and after years of efforts, the traditional construction techniques of Chinese wooden arch bridges were successfully inherited. Wooden arch bridge is a traditional wooden structure bridge widely used in mountainous areas of Fujian and Zhejiang, China. This type of Hongqiao technology, which was popular in the Northern Song Dynasty, is the tail end of China's ancient bridge-building techniques and the last rare living fossil in the history of bridge construction in China. In the context of modern civilization, the identity of wooden arched bridges as a means of transportation has declined. However, this beautiful building, like a rainbow drinking stream, has long corridors on the bridge that can shelter from wind and rain, market trade, and worship gods. People spend every grand festival on the funded construction of the corridor bridge, praying for the peace of the world and remembering their own joyful moments. Nowadays, behind the ancient corridor bridges of the Song, Yuan, Ming, and Qing dynasties, the new corridor bridges still have the ancient meaning of red, pitch black tiles, and the high and beautiful flying eaves. They are still nicknamed Que Bridge, Cuo Bridge, Rainbow Bridge, Wind and Rain Bridge, Centipede Bridge, Magpie Nest Bridge, and Shrimp Squirrel Bridge. The stunning scenery between the green mountains and waters is not only a skill, but also an indestructible spirit.

2.2 The principle behind the wooden arch bridges

The wooden arch bridge is made of raw wood materials, using traditional wooden construction tools and manual techniques, and utilizing core technologies such as “beam weaving”, shown in Figure 2. It does not require any iron pieces, but instead uses mortise and tenon joints, without nails, rivets, or bridge piers. Architectures only use rods of the same specifications, do not press, insert, or overlap them. Round logs are arranged vertically and horizontally, crossed and supported by each other, and extended section by section to form a complete wooden support-type main arch framework, shown in Figure 2. Not only that, wooden arch bridges have excellent compression performance. As long as the two ends of the arch are fixed and supported by their own strength, friction force, diameter, angle, horizontal distance, etc., they can withstand downward loads well. Taking the wooden arch bridge in the Qingming Riverside Map as an example, it can bear a weight of 2 to 3 tons.



Figure 2. A picture and a model of a wooden arch bridge.

Consider a simplified wooden arch bridge model, which is a symmetrical arch bridge with only three elements. Figure 3 shows its side view.

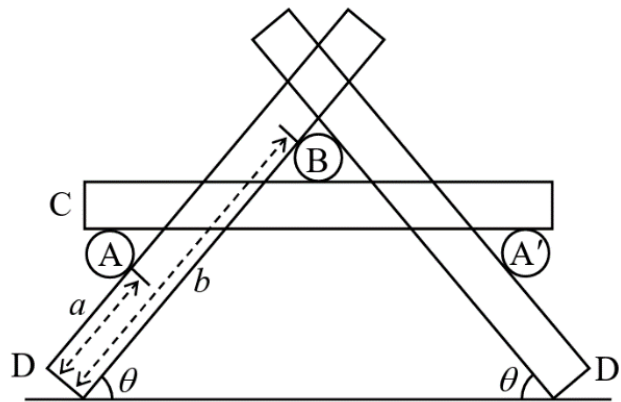


Figure 3. The side view of the simplified model.

Under the simplified model, only the balance of the arch bridge in the vertical plane (paper surface) is considered. Each pair of long rods parallel to the paper surface is equivalent to a rod with a mass of M . This wooden arch bridge can be further simplified into six rods: three short rods A, A', and B, and three long rods C, D, and D'. All rods are homogeneous cylinders with a radius of R , without grooves. The mass of the short rods is m , and the length of the long rods is L . The long rods do not touch each other. The surface of short rod B is smooth, and the static friction coefficient between the other rods is μ . The angle between the long rods D and D' and the horizontal ground is θ . The length between the lower end of rod D and the contact point with rod A is a , and the length between the lower end of rod D and the contact point with rod B is b . The magnitude of the gravitational acceleration is g .

Suppose $M = km$, $k = 10$, $R = 1 \times 10^{-1} \text{ m}$, $L = 5 \text{ m}$, $a = 1 \text{ m}$, and the lower end of rods D and D' is locked on the ground. We want to test the wood's roughness required to keep the bridge stable when θ varies. In other words, we want to calculate the smallest μ that can keep the system stable under different θ .

According to the geometrical relation between rod a and b, we have

$$b = a + 4R \cot \frac{\theta}{2}$$

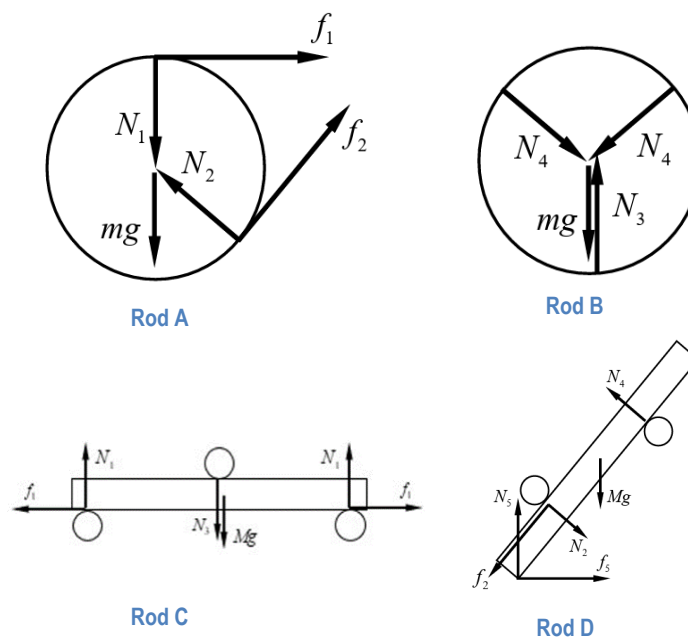


Figure 4. The force analysis of rods.

Because rod A is static, according to the force analysis shown in Figure 4, we can obtain the force balance equations:

$$\begin{aligned} N_1 + mg &= N_2 \cos\theta + f_2 \sin\theta, \\ f_1 + f_2 \cos\theta &= N_2 \sin\theta. \end{aligned}$$

Besides, we can obtain the torque balance equation:

$$f_1 R = f_2 R.$$

Because rod B is static, according to the force analysis shown in Figure 4, we can obtain the force balance equation:

$$N_3 = 2N_4 \cos\theta + mg.$$

Because rod C is static, according to the force analysis shown in Figure 4, we can obtain the force balance equation:

$$2N_1 = N_3 + Mg.$$

Because rod D is static, according to the force analysis shown in Figure 4, we can obtain the force balance equations:

$$\begin{aligned} N_5 + N_4 \cos\theta &= N_2 \cos\theta + f_2 \sin\theta + Mg, \\ f_5 + N_2 \sin\theta &= N_4 \sin\theta + f_2 \cos\theta. \end{aligned}$$

In addition, we can obtain the torque balance equation:

$$N_2 a + Mg \left(\frac{L}{2} \cos\theta - R \sin\theta \right) = N_4 b + f_2 2R.$$

According to the force balance equations and the torque balance equation of rod A, we can obtain

$$\begin{aligned} f_1 = f_2 &= \frac{\sin\theta}{1 + \cos\theta} N_2, \\ N_1 + mg &= N_2. \end{aligned}$$

From the force balance equations of rod B and rod C, we have

$$N_1 = N_4 \cos\theta + \frac{1}{2} (M + m)g.$$

By combining the equations above and the torque balance equation of rod D, we can obtain

$$(N_1 + mg)a + Mg \left(\frac{L}{2} \cos\theta - R \sin\theta \right) = \left[N_1 - \frac{1}{2} (M + m)g \right] \frac{b}{\cos\theta} + \frac{\sin\theta}{1 + \cos\theta} (N_1 + mg) 2R.$$

Furthermore, by solving this equation, we have

$$N_1 = \frac{mga + Mg \left(\frac{L}{2} \cos\theta - R \sin\theta \right) + \frac{1}{2} (M + m)g \frac{b}{\cos\theta} - \frac{\sin\theta}{1 + \cos\theta} mg 2R}{\frac{b}{\cos\theta} + \frac{\sin\theta}{1 + \cos\theta} 2R - a}.$$

From the equations above, we can also obtain

$$f_1 = f_2 = \frac{\sin\theta}{1 + \cos\theta} (N_1 + mg) = \frac{\sin\theta}{1 + \cos\theta} \times \frac{Mg \left(\frac{L}{2} \cos\theta - R \sin\theta \right) + \frac{1}{2} (M + 3m)g \frac{b}{\cos\theta}}{\frac{b}{\cos\theta} + \frac{\sin\theta}{1 + \cos\theta} 2R - a}.$$

To keep the bridge stable, the following inequations need to be satisfied:

$$f_1 \leq \mu N_1, \quad f_2 \leq \mu N_2.$$

Because $f_1 = f_2$ and $N_2 = N_1 + mg > N_1$, the second inequation will be satisfied under the premise of the first inequation. Therefore, only the first inequation needs to be satisfied.

According to the first inequation, we can obtain

$$\mu \geq \frac{f_1}{N_1} = \frac{\sin\theta}{1 + \cos\theta} \times \frac{Mg \left(\frac{L}{2} \cos\theta - R \sin\theta \right) + \frac{1}{2} (M + 3m)g \frac{b}{\cos\theta}}{mga + Mg \left(\frac{L}{2} \cos\theta - R \sin\theta \right) + \frac{1}{2} (M + m)g \frac{b}{\cos\theta} - \frac{\sin\theta}{1 + \cos\theta} mg 2R}.$$

The inequation above can be simplified to

$$\mu - \frac{\sin\theta}{1 + \cos\theta} \times \frac{k \left(\frac{L}{2} \cos\theta - R \sin\theta \right) + \frac{1}{2} (k + 3) \frac{b}{\cos\theta}}{a + k \left(\frac{L}{2} \cos\theta - R \sin\theta \right) + \frac{1}{2} (k + 1) \frac{b}{\cos\theta} - \frac{\sin\theta}{1 + \cos\theta} 2R} \geq 0,$$

where b can be replaced by $a + 4R \cot \frac{\theta}{2}$.

To calculate the required range of θ under a specific μ , I wrote a program called “Numerical Solver” which is shown in the appendixes by Python to help me calculate and show the value of expression visually by drawing the function

images.

In the code, I generated a function:

$$f(\theta) = \mu - \frac{\sin\theta}{1+\cos\theta} \times \frac{k\left(\frac{L}{2}\cos\theta - R\sin\theta\right) + \frac{1}{2}(k+3)\frac{b}{\cos\theta}}{a+k\left(\frac{L}{2}\cos\theta - R\sin\theta\right) + \frac{1}{2}(k+1)\frac{b}{\cos\theta} - \frac{\sin\theta}{1+\cos\theta}2R},$$

where b is replaced by $a + 4R\cot\frac{\theta}{2}$.

After that, the function graph is drawn and the required range of θ is calculated by using the extended library called “NumPy” from ANACONDA. The source code is simply an example with $\mu = 0.1$. Actually, μ can be changed to any value we want.

The function images under 4 different circumstances are shown in examples below:

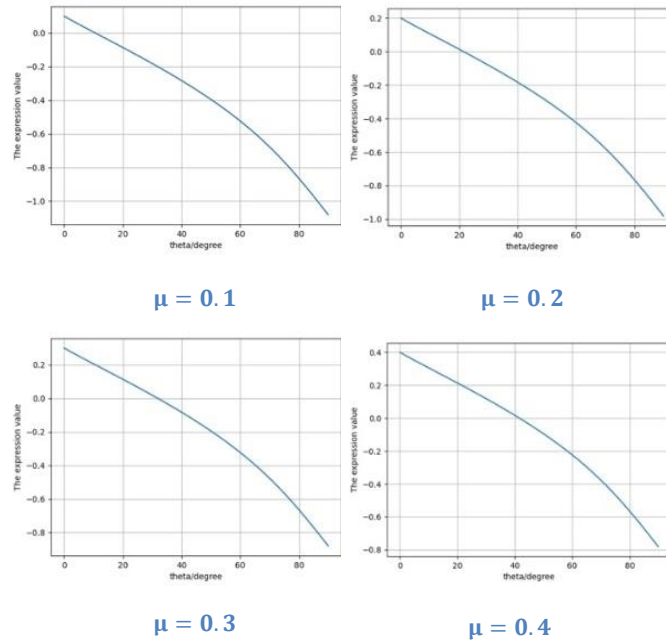


Figure 5. The function image under different μ .

According to the function images graphs and the outputs, we have:

- when $\mu = 0.1$, the required range of θ is $0^\circ \leq \theta \leq 10.576599^\circ$, which means when $\theta = 10.576599^\circ$, the required range of μ is $\mu \geq 0.1$;
- when $\mu = 0.2$, the required range of θ is $0^\circ \leq \theta \leq 21.441610^\circ$, which means when $\theta = 21.441610^\circ$, the required range of μ is $\mu \geq 0.2$;
- when $\mu = 0.3$, the required range of θ is $0^\circ \leq \theta \leq 31.871826^\circ$, which means when $\theta = 31.871826^\circ$, the required range of μ is $\mu \geq 0.3$;
- when $\mu = 0.4$, the required range of θ is $0^\circ \leq \theta \leq 41.583195^\circ$, which means when $\theta = 41.583195^\circ$, the required range of μ is $\mu \geq 0.4$.

To each θ , there is a minimum of required μ . Therefore, we can draw the image of the minimum of required μ and θ . Another program called “Function Image Generator” which is shown in the appendixes is created.

In this program, a function that denotes the minimum of required μ is defined:

$$g(\theta) = \frac{\sin\theta}{1+\cos\theta} \times \frac{k\left(\frac{L}{2}\cos\theta - R\sin\theta\right) + \frac{1}{2}(k+3)\frac{b}{\cos\theta}}{a+k\left(\frac{L}{2}\cos\theta - R\sin\theta\right) + \frac{1}{2}(k+1)\frac{b}{\cos\theta} - \frac{\sin\theta}{1+\cos\theta}2R},$$

where b is replaced by $a + 4R\cot\frac{\theta}{2}$ again.

$$g(\theta) = \frac{\sin\theta}{1+\cos\theta} \times \frac{k\left(\frac{L}{2}\cos\theta - R\sin\theta\right) + \frac{1}{2}(k+3)\frac{b}{\cos\theta}}{a+k\left(\frac{L}{2}\cos\theta - R\sin\theta\right) + \frac{1}{2}(k+1)\frac{b}{\cos\theta} - \frac{\sin\theta}{1+\cos\theta}2R}$$

Through NumPy, the function image of the minimum of required μ and θ is generated, which is shown in Figure 6:

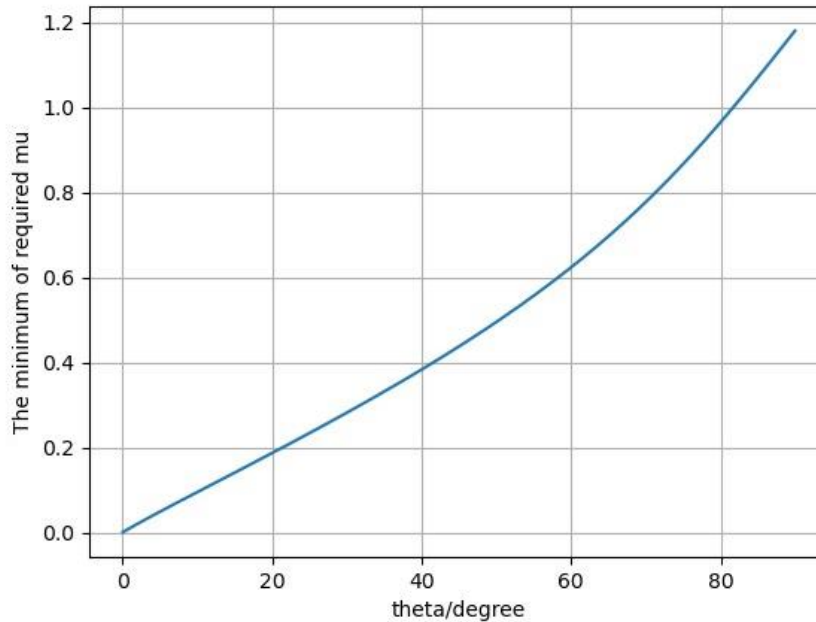


Figure 6. The function image of the minimum of required μ and θ .

Now, the required μ under different situations are clearly manifested and the influence of wood's roughness on the stability of wooden arch bridges is shown.

The above is only a calculation under a specific premise. Variables like k , R , L , and a can be changed to any reasonable values. I hope my calculation and programs will still be applicable under various circumstances and will provide some data for the architectures.

3. Understanding the Wisdom of Historical Wisdom and Protecting Them

The wooden arch bridge is not a cold and lifeless building, but an excellent skill created by the laboring people for thousands of years. It is a unique and vibrant spiritual force. However, in recent years, factors such as accelerated urbanization, scarcity of wood, and insufficient available building space have threatened the inheritance and survival of wooden arch bridge craftsmanship, and this tradition has faded. It is facing the threat of disappearing. I hope this paper can raise the awareness of people all over the world about protecting this amazing treasure and provide some applicable theoretical advice for architecture to revive wooden arch bridges in the future. I believe wooden arch bridges will show their unique charm again. See? Some people still don't give up protecting the bridges. Bridges stand up in their hands, continuing the life of this skill.

The world, on the old bridge, is nothing but the metabolism of time, just like the couplets on the Dragon Bridge: the past, present, present, and past are all ancient things, and the emptiness is the reality. The past is also the emptiness; The young newly-built corridor bridge aims to work with people to shield itself from the storms of time and lead to immortality. Look, the old man with his hands clasped in front of the shrine, his face filled with old piety in the ravines. You will believe in this immortal possibility: even though the deep mountains have already turned into a clear path, people still attach themselves to the clear water-covered bridge. This is not only a beautiful building to shield the wind and rain, but also a home to the world.

The wooden arch bridge is not only the prelude to the village landscape music but also the climax, representing the temperament and taste of the village. She is the "feng shui bridge" of the village, responsible for creating the village entrance and geography; She is the "landmark bridge" of the village, the cohesion and proud coordinate of the collective strength of the local people; She is also the "bridge of faith" in the village, carrying many devotions and prayers; She is still the "bridge of art", leaving behind many literary and literary talents and emotions; She is also a bridge between spirit and life, entangled with the homesickness of many distant wanderers. The innocence of childhood, the emotions of youth, the romance of youth, the passion of adulthood, and the tranquility of old age all remain in the long corridor of our hometown. The wooden arch bridge is a concrete expression of local culture, an endorsement of the thoughts and emotions of the mountain people, a carrier of rural literature and art, and a refuge for the lives and souls of the villagers. The construction technique of wooden arch bridges is rooted in this soil and is the technique of lifting the rainbow in the human world.

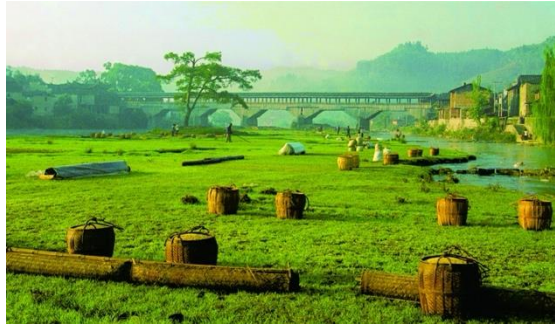


Figure 7. A bridge that leads people to a bright future.

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